How Students Explain and Teachers Respond

Ove Gunnar Drageset
UiT – The Arctic University of Norway
<ove.drageset@uit.no>

This article develops three different types of student explanations and studies how teachers respond to these. The data come from five classrooms at upper grade 5-7 (ages from eleven to thirteen) where all mathematics teaching for one week was filmed. These films were transcribed and student explanations identified. Through a close inspection of these, three categories of student explanations were developed. This enabled a closer study of how teachers respond. Typically, teachers respond by pointing out important details, by moving on without further comments, or by requesting students to provide more details.

In order to understand classroom communication in some depth there is a need for detailed concepts. This article will look into the IRE pattern (Cazden, 1988) and its limitations before moving on to more detailed and precise ways to characterise mathematical discourse in the classroom. Particularly, different types of student explanations are described and how teachers respond to these.

Theory

Perhaps the most cited pattern of classroom discourse is the IRE pattern (Initiate-Response-Evaluation) (Cazden, 1988). This describes a pattern where the teacher initiates a task or discussion, the student responds and the teacher evaluates. Franke, Kazemi, and Battey (2007) describe this pattern as a procedure-bound discourse with little emphasis on students' thinking and explanation. This means that IRE looks like a rather teacher dominated pattern where the student answers only when given permission and where the teacher controls the process (by choosing what needs an answer and when). It also looks like a pattern where the teacher decides what is right and wrong and by this maintains a position of authority given by a role instead of given by mathematical arguments. The road is relatively short from this to patterns of teacher domination described in the research literature. One such is the Topaze effect, which occurs in the situations when the teacher is so eager to get a specific answer that he gives hints to such an extent that the original task can be totally changed, or at least so that much less knowledge is required from the student (Brousseau & Balacheff, 1997). Another similar pattern is described by Lithner (2008) as guided algorithmic reasoning. This is the case where the teacher makes all the major decisions related to the process while the student contributes with all the easy parts, such as simple calculations. A third and related pattern is funnelling, described by Wood (1998) as the teachers using questions to funnel the conversation. The result is that most of the students' thinking is focused on trying to figure out the response the teacher wants instead of thinking mathematically. Topaze, guided algorithmic reasoning and funnelling all describe patterns where the teacher dominates the discourse by maintaining a tight grip and all could well fit into the IRE-pattern. These examples all could be used to argue that the IRE pattern is teacher dominated and consequently limits the students' opportunities to learn mathematics. But Wells (1993) uses examples from the classroom to illustrate how much variation is hidden within the IRE pattern. Most importantly, this variation includes

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 191–198. Sydney: MERGA.

qualitatively different initiatives, responses and evaluations. Also Cazden (2001) later emphasises that IRE includes more variation than first described. Consequently, within IRE there might be teachers dominating, but there also might be room for student contributions beyond answering teachers' questions and beyond evaluations limited to correct or in-correct. Whether the pattern can be described as IRE might not be the key to understanding qualities of a mathematical discourse. There are probably other factors that are more important. Mercer and Littleton (2007) argue that instead of looking at the number of questions a teacher asks one should look at the function of these questions. Even though many teachers control the initiation and evaluation in the IRE pattern there might be differences in how this is enacted that results in different functions. To understand the function of initiations and evaluations one has to study them as part of the dialogue; probably in more detail than given in the IRE pattern in order to differentiate between different types of teacher initiations, student responses and teacher evaluations.

One such approach of describing classroom discourse in more detail is provided by Fraivillig, Murphy, and Fuson (1999) with the framework called ACT (Advancing Children's Mathematics). The framework is developed based on an in-depth analysis of one skilful teacher and has three components: eliciting children's solution methods, supporting children's conceptual understanding and extending children's mathematical thinking. Based on the examples from Wells (1993), it seems to be possible to both elicit, support and extend children's mathematical thinking within the IRE pattern. Another detailed approach is provided by Alrø and Skovsmose (2002) and their eight communicative features: getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging and evaluating. It is possible to fit most of these concepts into an IRE pattern. A third detailed approach is the model suggested by Mortimer and Scott (2003) to describe how teachers work with students to develop ideas along two dimensions: the dialogic - authoritative and the interactive - non-interactive dimension. The dialogic-authoritative dimension is especially interesting when related to IRE and where the authority is placed during the evaluation. Even though the teacher leads the evaluation in an IRE pattern, it is possible to let the mathematical arguments be the authority and include students' arguments in a more dialogic approach. On one hand, most patterns described by ACT (Fraivillig et al., 1999), the eight communicative features (Alrø & Skovsmose, 2002) and the two dimensions from Mortimer and Scott (2003) might be found in an IRE pattern. On the other hand, it is more likely to find a higher frequency of for example extending, challenging and dialogic/interactive patterns in other types of practices, such as in instructive communication (Brendefur & Frykholm, 2000) or in an inquiry/argument classroom (Wood, Williams, & McNeal, 2006).

A recent approach of describing classroom discourse in mathematics in detail is the redirecting, progressing and focusing framework (Drageset, 2014) and the corresponding description of five different types of student comments (Drageset, 2013). These were developed from a study of five practices where conversation analysis was used to study the mathematical discourse on a turn-by-turn basis. The redirecting, progressing and focusing framework (see Table 1) describes thirteen different teacher actions during classroom discourse and organises these in three groups or superordinate categories.

Table 1
The Redirecting, Progressing and Focusing Actions Framework

Redirecting actions	Progressing actions	Focusing actions
Put aside	Demonstration	Enlighten detail
Advising a new strategy	Simplification	Justification
Correcting question	Closed progress details	Apply to similar problems
	Open progress initiatives	Request assessment from other students
		Notice
		Recap

The redirecting actions are typically when the teacher wants the student(s) to change their approach. This was done by putting aside the student suggestion, by advising a new strategy, or by asking questions in such a way that it included a correction. The progressing actions are about moving the progress forward. This was done either by demonstrating the entire solution process, by simplifying through hints and suggestions, by asking closed and often basic questions to move along one step at a time while the teacher controlled the process, or by asking open questions and leaving it to the student(s) to choose how to progress. The focusing actions are about stopping the progress to look deeper into some important detail and consist of two main types. One type is to request students either to enlighten in detail how they solved or thought to arrive at the answer, to justify why their answer or method was mathematically correct, to apply the method on a similar problem, or to assess. The other type is the teacher pointing out important ideas or rules either during the solution process (notice) or after the solution was found or agreed upon (recap).

Using the same data, Drageset (2013) also developed five categories of student comments: explanations, initiatives, teacher-led responses, unexplained answers and partial answers. These add to the categories describing teacher actions and together give a set of concepts able to describe all mathematically related comments in these five practices.

The frameworks describing teacher and student comments on a turn-by-turn basis (Drageset, 2013, 2014) were developed from five practices that could all be labelled as consequently using the IRE pattern. The thirteen categories of teacher comments and five categories of student comments illustrate how large a variation might be hidden within IRE.

Research Question

IRE only offered three concepts usable to describe mathematical discourse: initiation, response, and evaluation; and a practice is described as either IRE or not IRE. More detailed frameworks such as ACT (Fraivillig et al., 1999), the eight communicative features (Alrø & Skovsmose, 2002) and the combined framework describing teacher and student comments (Drageset, 2013, 2014) enable us to go one step deeper. With more concepts that can be tools to describe communication on a turn-by-turn basis it becomes possible to look at how these concepts or categories are related to each other in different ways and in different practices. For example, it becomes possible to inspect how teachers respond to particular types of student comments in order to describe different qualities of the discourse. The aim of this article is to first describe different types of student explanations and then inspect how teachers respond to these. The research question is this: What different types of student explanations exist, and how do the teachers respond to student explanations?

Method

This study is part of a larger study of 356 teachers that answered a test of their mathematical knowledge for teaching and a questionnaire about their beliefs. Based on the results, five teachers were picked for further study. These were all teaching at upper primary (grade five to seven, mainly ages 11 to 13) and had between five and twenty-five years of experience as mathematics teachers. The majority of teachers in Norwegian primary schools are educated as general teachers, typically with some education in mathematics. These five all belonged to this majority. A researcher visited the classrooms and filmed all mathematics teaching for one week, typically four or five lessons of 45 minutes. The filming started at the first lessons of the topic of fractions that school year. The camera focused on the teacher and the microphone was able to catch everything the teacher said and almost everything said to the teacher.

All the films were then transcribed and analysed by looking at single turns and characterising them regarding their role in the dialogue. This means that the single turns were not analysed in isolation but as a part of the dialogue. As the analysis described the dialogue in itself and not as a tool to see something else, it belongs to conversation analysis. Similar turns were put into groups and formed initial categories and through a rather long process of defining, redefining, merging and splitting groups a framework that was able to describe all mathematically related turns was developed (for further details, see Drageset, 2013, 2014).

The data analysis in this article builds on the work done during the development of the framework. As all student explanations were categorised (and marked) it became possible to go deeper into the data by re-visiting all student explanations and studying how the teachers responded to each of them. Overall in these five practices, one of eight student comments were explanations. More than half of these were about explaining action (what and how) while explaining reasons and explaining concepts were just over and under a quarter.

Findings

Three Types of Student Explanations

The study of single student turns developed five superordinate categories: explanations, partial answers, student initiatives, teacher-led responses and unexplained answers (Drageset, 2013). This article focuses on student explanations and the superordinate category of explanations includes student explanations. The following excerpt includes two rather similar explanations:

Student:	It is several different fractions which has different denominator, but … means the same nevertheless.
Teacher:	Different denominators but means the same nevertheless, how would you clarify that?
Student:	Um, it is divided in more pieces but it is the same amount … is divided in more.

Both these student comments are about explaining the concept of equivalent fractions. By grouping student explanations related to concepts a basic category was formed. Most such explanations of concepts were students trying to explain the concepts of fractions, numerators and denominators. Quite often, the explanations were unpolished like the example above, where the students struggled to find precise expressions.

Another type of explanation were related to reason:

Student: One sixth of eighteen equals three.
Teacher: Why?
Student: Because one … three times six are eighteen.

Here the student tries to explain why he knows that one sixth of eighteen equals three. In the basic category of explaining reason there are many different explanations, more or less complete and more or less mathematically founded. By grouping all explanations that have in common that the student tries to justify by explaining why an answer or a method is correct another basic category was formed.

But all explanations could not be described as explaining concept or reason, such as this one:

Student: Then he gives one fourth of the remaining to his sister.
Teacher: Okay, what do you have to do now then?
Student: Then I have to take one fourth of one hundred which is twenty-five because twenty-five multiplied by four are … (impossible to hear). And then … one hundred minus twenty-five, that is seventy-five.

This is a description of the steps of the solution process, not trying to justify why the method is correct or what the concepts involved means. This category consists of explanations about how or what, how to reach a solution or what to do, and is named explaining action. Often, these explanations are referring to a method to find an answer by explaining either what or how something can be done (before doing it) or was done (after doing it). These formed the basic category of explaining action.

As illustrated by Drageset (2013), the superordinate category of student explanations is quite precisely defined and differs from the other categories of student comments. The above examples illustrates that it is possible to go one step deeper and find three basic categories within the superordinate category of explanation and that these also are rather distinct: Explaining concept, Explaining reason, and Explaining action. The different explanations were typically requested by the teacher and gave explicit details about concepts, reasons and actions. Such explanations seem to serve both as a control of a student's understanding and as a way to make details explicit in order to share knowledge. In the following they will be inspected individually to see how the five teachers typically responded to each type.

Teachers' Response to Student Explanations

During a discussion about fractions equal to one half, the teacher asks the students to find the denominator when the numerator is 34 and the fraction has to be equal to one half.

Student: Sixty-eight.
Teacher: Bravo. Sixty-eight (writes the fraction on the blackboard). Because … what was the reason for this?
Student: Because three plus three is six and four plus four is eight.
Teacher: Yes. Double. Yes. Double the denominator related to the numerator.

The student explains the reason in a rather algorithmic way and the teacher responds to this by pointing out and clarifying the general idea before the process continues with similar tasks using the idea pointed out by the teacher. Typically, teachers point out an important idea or rule during a solution process or between similar tasks so that the idea or rule can be used immediately by the students. Such responses are called *notice* (Drageset, 2014) and are the most frequent, following students' explaining reason.

At other times the teacher response to students explaining reason looked like this:

Student 1: One sixth of eighteen is three.
Teacher: Mmm (confirming). What is three sixths of eighteen?
Student 1: Ehm … I don't know
Teacher: But if one sixth is three … (other students comment omitted).
Student: Nine.
Teacher: Yes, but why?
Student: Because it becomes more. Three, six, nine.
Teacher: Three, six, nine, yes. One sixth of thirty?

In this case the teacher asks why an answer is correct, accepts the answer and goes on without further comments or clarification. The difference between the two excerpts above goes into the core of orchestrating; when to go into details and when to move on. While pointing out, emphasising and clarification are important teacher actions in order to help other students understand, it also seems obvious that a teacher cannot go into details about every explanation. Then there would be little progress. Also, some student explanations might be assessed as sufficient and then the teacher most likely sees no need for an intervention.

Looking at student explaining concepts, this is an example of a frequent response:

Teacher: What does it mean to find equivalent fractions? Anyone that can say something about it? What do you do then?
Student: There are many different fractions that have different denominators, but … means the same anyway.
Teacher: Different denominators but means the same anyway, how would you clarify that?

The student does not really answer the initial question from the teacher. But instead of keeping the focus on how it is possible to find equivalent fractions the teacher follows up the student comment with a new question that requests a clarification. This is also a key question about equivalent fractions; how can fractions be different and equal at the same time? This is about addressing the important details and make their reasons visible and understandable for every student. Different teacher actions to clarify details, rules, reason, concepts and ideas were observed quite frequently, either by the teacher clarifying or by the teacher requesting students to clarify.

An example of a teacher response to students explaining actions is this one:

Teacher: Three tenths and twenty-nine hundredths. Can you manage that one? Which one is the largest?
Student 1: Three tenths.
Teacher: You think that it is three tenths? How did you manage … It is completely correct, but how did you think then? How did you manage to solve it?
(Student 1 does not answer the question so the irrelevant responses are omitted)
Student 2: Because twenty-nine hundredths becomes twenty-nine parts of a hundred, while thirty, no, three tenths becomes thirty hundredths.
Teacher: Precisely. Three tenths is the same as thirty hundredths and that is larger than twenty-nine hundredths.

Student 2 gives an explanation of how to find the solution. The teacher repeats the student explanation, but also changes it a little bit to emphasise that three tenths is the same as thirty hundredths and that is the larger one. This pointing out, by recapitalising the important idea, concludes the discussion.

Discussion

The three types of student explanations refer to different types of mathematical work that are important. Explaining action is about sharing the way a solution was found and such explanations are important in order to help teachers assess and fellow students follow the line of thought. Explaining action might be rather procedural, using standard methods or rules and explaining each step in the particular case. Explaining reason is clearly different as it goes into the reason why a rule or a method is a mathematically justified choice in this case, or why an answer is correct. Explaining reason is a type of mathematical work that goes into the core of mathematical understanding. While explaining action and reason typically is about solving tasks, explaining concept is different. It is about explaining concepts that students need to understand in order to solve tasks in a meaningful way.

Altogether for all three types of explanations, the teachers responded with focusing actions in almost two of every three cases. The main type of teacher response was to use notice, which is about pointing out rules and reasons, especially during solution processes but also during other types of discourse. The purpose of using notice seems to be to clarify for other students to follow the thoughts or to help the students back on track. Another major type was to use closed progress details where the main objective seems to be to move the process forward without going into further detail. This is a natural thing to do if the teacher assesses the student explanation to be both mathematically correct and clear enough for other students to understand. Arguably, every mathematics lesson needs some progress. In addition to this, requesting students to enlighten details was the third most frequent teacher response. This is a request for the students to explain how a solution was reached or what to do in order to find the solution. Making details available and visible for other students is, according to Franke et al. (2007), one of the most powerful moves a teacher can take.

There were not any major differences in how the teacher responded to the three different types of student explanations (explaining reason, explaining concept, explaining action). The only finding to report was that teacher responses to students explaining reason and concept more often where focusing actions (two of three) than responses to explaining action (one of two). This might indicate that students' explanations of reasons and concepts more often needed further clarification than explanations of action needed.

Conclusion

The development of concepts to describe mathematical discourse is important in order to describe what happens in the classroom in more detail. The IRE pattern (Cazden, 1988) only gives us three concepts to describe a discourse where the teacher talks every second time (the teacher comments are either initiatives or evaluations and the student comments are responses). More detailed frameworks such as the ACT (Fraivillig et al., 1999), the eight communicative features (Alrø & Skovsmose, 2002) and turn-by-turn based frameworks of teacher and student comments (Drageset, 2013, 2014) provide us with concepts that enables us to describe variation in much more detail, both within IRE and in other practices. Wells (1993) has illustrated how too general concepts, such as IRE, might hide more information than they give. The three distinct types of student explanations add to the work of developing more detailed tools in order to describe variation. The aim is not to describe variation for its own case; it is to characterise different qualities of communication in order to understand more of the learning process.

This study is limited in the sense that only five practices were filmed, all being from Norway and all typical IRE-practices. Even though most concepts would probably be capable of describing other practices, there are probably other types of teacher and student comments in other cultures or other types of practices. Consequently, there is a need for further research in order to describe and develop concepts to describe comments or patterns not found in these five classrooms.

References

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht: Kluwer Academic Publishers.

Brendefur, J., & Frykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3*(2), 125-153. doi: 10.1023/a:1009947032694

Brousseau, G., & Balacheff, N. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Cazden, C. B. (1988). *Classroom discourse: the language of teaching and learning*. Portsmouth: Heinemann.

Cazden, C. B. (2001). *Classroom discourse: the language of teaching and learning*. Portsmouth: Heinemann.

Drageset, O. G. (2013). *Different types of student comments in the mathematics classroom.* Unpublished manuscript.

Drageset, O. G. (2014). Redirecting, progressing, and focusing actions—a framework for describing how teachers use students' comments to work with mathematics. *Educational Studies in Mathematics, 85*(2), 281-304. doi: 10.1007/s10649-013-9515-1

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education, 30*(2), 148-170.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256): Reston, VA: National Council of Teachers of Mathematics.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255-276.

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. London: Routledge.

Mortimer, E. F., & Scott, P. (2003). *Meaning making in secondary science classrooms*. Buckingham: Open University Press.

Wells, G. (1993). Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and Education, 5*(1), 1-37.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: National Council of Teachers of Mathematics.

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education, 37*(3), 222-255.